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THE EFFECT OF VISCOUS AND ELASTIC CONTROL SYSTEM RESTRAINTS ON HELICOPTER HOVERING STABILITY AND CONTROL

PART I - THEORETICAL ANALYSIS

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TABLE OF CONTENTS

1.	SUMMARY	1
2.	INTRODUCTION	2
3.	DISCUSSION OF RESULTS	5
4.	CONCLUSIONS	10
5.	LIST OF SYMBOLS	12
6.	THEORETICAL ANALYSIS	15
7.	FIGURES	21
8.	REFERENCES	34

1. SUMMARY

A stabilization system is considered in which the blade mass and aerodynamic characteristics are modified so as to result in cyclic feathering moments tending to stabilize the helicopter. The swash plate is permitted to respond to these stabilizing moments by being connected to the pilot's cyclic pitch control by a spring and a viscous damper in parallel. The effects of incorporating different types of mass and aerodynamic characteristics and different magnitudes of elastic and viscous restraints are considered. A theoretical analysis is presented by means of which root loci and response curves are obtained. An evaluation of the usefulness of the device is attempted.

2. INTRODUCTION

While it appears feasible to successfully stabilize a helicopter with an automatic pilot, the incorporation of such a device imposes a severe penalty when factors such as weight, cost, and maintenance are taken into consideration. The utilization of a simple mechanical stabilization method, such as the one considered in this study, could decidedly reduce such penalties in addition to minimizing the possibilities of failure in the system.

The stabilization method under consideration was previously proposed by R.H. Miller (Ref. 1 and 2). In this system the pilot's cyclic pitch control is connected to the non-rotating part of the swash plate through a spring and a viscous damper in parallel (Fig. 1). Harmonic moment variations about the blade feathering axis are fed back through the blade linkages to the swash plate. The motion of the swash plate in response to these moment variations introduces cyclic pitch changes which, for a properly designed system, have a stabilizing effect upon the helicopter. The harmonic moment variations can be obtained by displacing the blade center of gravity or the blade aerodynamic center from the feathering axis, or by use of an unsymmetrical airfoil section. Since the restraints are applied to the non-rotating part of the swash plate, different degrees of stabilization are obtainable in the lateral and longitudinal motions. It would thus be possible to provide adequate stabilization for the pitching motion without overstabilizing the helicopter in roll.

Some of the effects of this means of stabilization can be envisioned as follows. If the tip path plane of a helicopter is rotated with respect to the horizon about the pitching axis, gyroscopic moments proportional to the pitching velocity act on the blades about the rolling axis. Since the pitching motion referred to is with respect to the horizon, it can be envisioned as combining the pitching motion of the helicopter and the pitching of the tip path plane with respect to the shaft. If the blade center of gravity is located ahead of the blade feathering axis and the swash plate is allowed a degree of freedom, these gyroscopic moments will increase the incidence of blades advancing in the direction of downward tip path plane pitching and decrease the incidence of retreating blades. This cyclic feathering action increases the damping in pitch of the helicopter by damping the pitching motion of the tip path plane. Such a damping of the tip path plane with respect to its initial plane of rotation has an adverse effect upon the response of a helicopter to a control input but the lag of the tip path plane provides moments about the helicopter center of gravity which have a stabilizing effect. It can readily be seen that if the blade aerodynamic center is located behind the feathering axis the rotor's inherent damping in pitch will be increased by a factor dependent upon the amount of aerodynamic center offset since the asymmetrical airload moments will produce a cyclic pitch change tending to oppose the rotor's pitching motion. This offset of the blade aerodynamic center also results in a control feedback proportional to the translational velocity of the helicopter as does the use of cambered blades. This feedback, however,

has a small effect in most cases. It should be noted that if the ratio of aerodynamic unbalance or blade pitching moment coefficient to elastic restraint is too great, an instability can occur.

Because of the large number of variables involved, an analog computer was employed to obtain the velocity responses. Since the characteristics of the motion can be demonstrated from either the attitude or the velocity response, the latter was utilized because it permitted some simplification in analog operation.

This report covers the first phase of an investigation which will conclude with the installation on a helicopter model of the stabilization device described above. For the purposes of later comparison with flight test data the results presented in this paper were obtained for the model to be tested. The general results and trends should be equally valid for a full scale helicopter.

3. DISCUSSION OF RESULTS

If the small feedback proportional to horizontal velocity is neglected, and if the pitching velocity of the tip path plane with respect to the horizon is considered equal to the pitching velocity of the helicopter, the system under consideration is equivalent to an autopilot with feedback proportional to the helicopter pitching velocity and a time lag equal to the ratio of viscous to elastic restraint. Under these assumptions, Evans' root locus method (Ref. 3) can be applied to the system (Fig. 2 and 3). If the time lag is zero, the system performs like a pure rate gyro. The locus of roots (Fig. 3, $\frac{c_2}{k_2} = 0$) shows that for zero gain the response is that of the unstabilized helicopter. As the feedback magnitude is increased from the zero value, the damping of the long period motion improves while the period of oscillation increases. For this condition, however, the helicopter cannot be made more than neutrally stable. The short period oscillation is heavily damped in all cases and has a negligible effect on the response. As the time lag is increased, better stability characteristics are obtainable. It should be noted that except for very large values of time lag, increasing the gain not only stabilizes the helicopter but increases the period of the long period oscillation. This implies that the rate of response is decidedly decreased from that of the unstabilized helicopter. For the very large values of time lag for which the period of oscillation decreases for an initial increase in feedback (Fig. 3, $\frac{c_2}{k_2} = \infty$), the predominant motion will be a slow aperiodic convergence which will prevent the initial rate of response from being as great as that of the

unstabilized helicopter. The magnitude of the feedback is proportional to the amount of blade center of gravity or aerodynamic center offset and inversely proportional to the amount of viscous damping in the restraint.

For any degree of viscous damping in the restraint a very large spring constant produces a very stiff restraint and the response of the helicopter to any control motion approaches that of the unstabilized configuration (Fig. 4 to 6). As the spring stiffness is decreased the damping of the long period oscillation becomes greater with the effect upon the frequency dependent upon the amount of viscous damping in the restraint. If the spring stiffness is maintained at a constant value while the amount of damping is varied, it is found that a large viscous restraint approaches the unstabilized condition. As the amount of viscous damping is lessened, the frequency of the long period motion is decreased with the effect upon the damping of the oscillation varying according to the magnitude of the elastic restraint (Fig. 7 to 9).

If an offset weight on an otherwise mass balanced blade is used to obtain the necessary feedback of forces to the swash plate, the amount of feedback is proportional to the product of the radial position, the chordwise position, and the mass of the weight. An increase in any of these parameters increases the feedback of the system (Fig. 10 and 11).

Whereas the assumption was made in obtaining the root loci that all feedbacks except that proportional to pitching velocity of the tip path plane with respect to the horizon were negligible, this assumption was not

used in obtaining the velocity response curves. The agreement in the results suggests that the assumption is justified and results in a negligible error.

The root loci and response curves were obtained for a model helicopter with the following characteristics:

$$\theta_c = .1710 \text{ rad.}$$
$$\Omega = 120.4 \text{ rad. sec.}^{-1}$$
$$R = 3 \text{ ft.}$$
$$s = 5.75$$
$$c = .208 \text{ ft.}$$
$$\gamma_1 = 2.992$$
$$\delta = .0246$$
$$T = 34.13 \text{ lb.}$$
$$h = .943 \text{ ft.}$$
$$e = 0$$
$$I_{y_0} = .206$$
$$\bar{m} = 1.357 \text{ slugs}$$
$$\beta_0 = .0436 \text{ rad.}$$
$$K_s = .077$$
$$b = 2$$

If cambered blades are utilized it should be noted that too large a ratio of blade pitching moment coefficient to elastic restraint can cause a longitudinal divergence of the swash plate from the trim position. Similarly an excessive blade aerodynamic center effect can have the same result. It would therefore not be advisable to make any installation without a preliminary theoretical investigation.

An evaluation of the usefulness of this method of stabilization is dependent upon the type of control response desired. Increasing the stability of the helicopter results in a decrease in the rate of response. If it is desired to maintain the rate of response of the unstabilized helicopter, this system cannot be used by itself. In order to keep the original rate of response and yet have a well damped oscillation, the frequency of the long period motion must be increased greatly without allowing other modes to become more poorly damped. This perhaps could be accomplished by a system investigated by Miller (Ref. 2) in which an additional feedback dependent upon the motion of the tip path plane with respect to the helicopter was included. This was not considered in the present investigation due to the difficulties in physically incorporating such a linkage in a conventional rotor system.

In any small helicopter in which the rate of response is so large as to be undesirable, this means of stabilization appears to have great promise. It would perform the dual task of reducing this high rate of response and also imparting a high degree of stability to the helicopter. The fact that installation could be made at a minimum weight and cost penalty also appears highly valuable.

It may be desirable to have different degrees of stability in stick-fixed and stick-free flight. A stiff combination of viscous and elastic restraints between the pilot's control and the swash plate would insure a rapid rate of response to a control input. If then a comparatively soft combination of restraints were placed between the pilot's control and the

frame of the helicopter, a high degree of stability could be obtained in stick-free flight.

In large helicopters, any loss in rate of response usually cannot be tolerated. In addition, any increase in already large stick forces by incorporation of mass or aerodynamic unbalance appears unadvisable. This means of stabilization therefore appears to have little application to helicopters of larger size.

4. CONCLUSIONS

1. By proper utilization of blade mass overbalance, aerodynamic underbalance, and camber, and by introducing suitable viscous and elastic restraints between the cyclic pitch control and the non-rotating part of the swash plate, stabilization of all modes of the motion of a helicopter can be achieved.

2. The possibility exists of making a helicopter extremely unstable by improper use of the methods discussed here. A theoretical analysis would therefore be necessary before attempting any installation to take into account the peculiar characteristics of the given configuration.

3. Since it was found that limitations exist as to the rate of response obtainable when the helicopter is stabilized, an evaluation of this stabilization method is dependent upon the type of response desired. In small helicopters in which the rate of response is very large, the system provides a method of reducing this undesirable characteristic while making the motion stable. Such a system would furthermore be desirable from the standpoint of weight, cost, and maintenance. Due to the decrease in rate of response inherent in the system and the necessary increase in magnitude of stick forces, this method of stabilization does not seem applicable to large helicopters.

4. Further study involving model testing will reveal many of the difficulties which might arise in an installation on a full scale helicopter. Such a model testing program will also serve as a check on the theoretical analysis.

5. The general trend indicated in this paper should remain the same for any conventional full scale helicopter, however, because of the change in location of the roots on the complex plane, the shape of the root locus curves (Fig. 3) may be modified.

5. LIST OF SYMBOLS

A_1, B_1 = lateral and longitudinal components of swash plate inclination with respect to shaft, rad.

A_{1c}, B_{1c} = lateral and longitudinal components of cyclic pitch control position with respect to shaft, rad.

C_h = $\frac{1}{2} a \rho c^2 \Omega^2 R^3 h$.

C_{m_o} = blade element pitching moment coefficient

D = differential operator

I_1 = blade moment of inertia about the flapping hinge = $\int_0^R mr^2\,dr$

I_{y_o} = moment of inertia of helicopter in pitch, lbs. ft. sec.^{2}

M'_s = $\int_0^R mrcd_1\,dr$

$M'_{y_{a_1}}$ = $-\frac{16}{\gamma_1 \Omega}$

$M'_{y_{\mu_x}}$ = $2\left(\frac{2}{3}\theta_0 + \lambda_0\right)$

R = radius of rotor, ft.

T = rotor thrust, lbs.

U_P, U_T = normal and tangential velocities at a blade element, ft. sec.$^{-1}$

a = blade element lift curve slope, rad.$^{-1}$

a_1, b_1 = longitudinal and lateral components of β, rad.

b = number of blades

c = blade chord, ft.

c_2 = viscous restraint about blade feathering axis, ft. lb. sec. rad.$^{-1}$

d_1 = blade c.g. location ahead of feathering axis, percent chord

e = distance of flapping hinge from rotational axis, ft.

h = distance from helicopter c.g. to hub, ft.

h_1 = blade a.c. location ahead of feathering axis, percent chord

k_2 = elastic restraint about blade feathering axis, ft. lb. rad.$^{-1}$

l_1 = offset mass position ahead of blade feathering axis, ft.

m = blade mass, slugs ft.$^{-1}$

$\bar{m}$ = helicopter translational mass, slugs

m_1 = offset mass, slugs

r = distance from rotational axis to blade element, ft.

r_1 = distance of offset mass from rotational axis, ft.

v = rotor induced velocity (assumed constant over disc)

$x_{m_1}, y_{m_1}, z_{m_1}$ = coordinates of offset mass referred to fixed axes, ft.

x_b, y_b, z_b = coordinates of blade element referred to fixed axes, ft. (Fig. 12)

x_o, y_o, z_o = coordinates of rotor hub referred to fixed axes, ft. (Fig. 12)

α_1, α_2 = helicopter pitch and roll angles referred to fixed axes, rad. (Fig. 12)

β = blade flapping angle referred to fixed axes (Fig. 12)
$= \beta_o - a_1 \cos\psi - b_1 \sin\psi$

β_o = blade coning angle, rad.

$$\gamma_1 = \frac{c \rho a R^4}{I_1}$$

δ = mean blade profile drag coefficient

θ = blade pitch angle referred to fixed axes, rad.
$= \theta_o - \theta_1 \cos\psi - \theta_2 \sin\psi$

θ_o = blade collective pitch angle, rad.

Θ_1 = lateral component of Θ, rad.
$= A_1 - \alpha_2$

Θ_2 = longitudinal component of Θ, rad.
$= B_1 + \alpha_1$

λ_a = mean inflow factor = $-\dfrac{v}{\Omega R}$

$\mu_x = \dfrac{\dot{x}_o}{\Omega R}$

ρ = air density, slugs ft.$^{-3}$

ψ = blade azimuth position (Fig. 12)

Ω = rotor speed, rad. sec.$^{-1}$

6. THEORETICAL ANALYSIS

If a mass m_1 is attached to the blade at a radius r_1 and at a distance l_1 from the feathering axis, the moment of this mass about the feathering axis can be expressed by

$$(M_x)_{m_1} = l_1 m_1 \left[r_1 \ddot{\beta} + l_1 \ddot{\theta} + (\ddot{x}_{m_1} \cos\psi + \ddot{y}_{m_1} \sin\psi)\beta \right.$$
$$\left. + (\ddot{y}_{m_1} \cos\psi - \ddot{x}_{m_1} \sin\psi)\theta \right]$$

where

$$x_{m_1} = x_0 - r_1 \cos\psi + l_1 \sin\psi$$
$$y_{m_1} = y_0 - r_1 \sin\psi - l_1 \cos\psi$$

If the blade center of gravity without the attached mass is located at a distance $d_1 c$ ahead of the feathering axis (Fig. 13), the blade inertia moments about the feathering axis can be written as

$$(M_x)_m = \int_0^R [\ddot{z}_b + (\ddot{x}_b \cos\psi + \ddot{y}_b \sin\psi)\beta] m d_1 c \, dr$$

where

$$x_b = x_0 - r \cos\beta \cos\psi \cong x_0 - r\cos\psi$$
$$y_b = y_0 - r \cos\beta \sin\psi \cong y_0 - r \sin\psi$$
$$z_b = r \sin\beta \cong r\beta$$

$$(M_x)_m = [\beta_0 \Omega^2 + (2\ddot{a}_1 \Omega - \ddot{b}_1) \sin\psi + (-2\dot{b}_1 \Omega - \ddot{a}_1)\cos\psi] M_s'$$
$$+ [\ddot{y}_0 \beta_0 \sin\psi + \ddot{x}_0 \beta_0 \cos\psi] \bar{M}_s'$$

$$M'_s = \int_0^R m c d_1 r \quad dr$$

$$M'_\alpha = \int_0^R m c d_1 \quad dr$$

With the blade aerodynamic center at a distance $h_1 c$ ahead of the feathering axis (Fig. 13), the aerodynamic moments about the feathering axis are

$$(M_x)_h = -\frac{1}{2}\rho\int_0^R c^2\left[a\left(U_T^2\theta + U_P U_T\right)h_1 + C_{m_0} U_T^2\right] dr$$

where

$$U_P = -\dot{z}_b \cos\beta - (\dot{x}_b \cos\psi + \dot{y}_b \sin\psi)\sin\beta + \lambda_a \Omega R$$

$$= (r\Omega b_1 + r\dot{a}_1 - \beta_0 \dot{x}_0)\cos\psi$$
$$+ (-r\Omega a_1 + r\dot{b}_1 - \beta_0 \dot{y}_0)\sin\psi + \lambda_a \Omega R$$

$$U_T = \dot{x}_b \sin\psi - \dot{y}_b \cos\psi$$

$$= \dot{x}_0 \sin\psi - \dot{y}_0 \cos\psi + \Omega r$$

$$(M_x)_h = -\frac{1}{2}\rho c^2 \Big\{ a\Big[\theta_0\Omega^2\frac{R^3}{3} + \lambda_a \Omega^2 \frac{R^3}{2}$$

$$-\Big(-2\Omega\theta_0 \dot{y}_0 \frac{R^2}{2} - \Omega^2\theta_1\frac{R^3}{3} + \Omega^2 b_1 \frac{R^3}{3} + \Omega \dot{a}_1 \frac{R^3}{3}$$

$$-\beta_0 \dot{x}_0 \Omega \frac{R^2}{2} - \lambda_a \Omega R^2 \dot{y}_0\Big)\cos\psi$$

$$+\left(\Omega R^2\theta_o \dot{x}_o - \Omega^2 a_o \frac{R^3}{3} - \Omega^2 a_1 \frac{R^3}{3} + \Omega b_1 \frac{R^3}{3} - \beta_o \dot{y}_o \Omega \frac{R^2}{2}\right.$$

$$\left. + \lambda_a \Omega R^2 \dot{x}_o\right) \sin\psi \Big] h_1 + C_{m_o}\Big[\frac{\dot{x}_o^2}{2} R + \frac{\dot{y}_o^2}{2} R$$

$$+ \Omega^2 \frac{R^3}{3} - \Omega R^2 \dot{y}_o \cos\psi + \Omega R^2 \dot{x}_o \sin\psi \Big]\Big\}$$

The moment about the feathering axis due to the elastic and viscous restraints in the control system can be given

$$(M_x)_d = -\frac{\left[(B_{1c} - B_1)(k_2 + Dc_2)\right]}{\sin\psi}$$

The summation of all moments about the feathering axis must equal zero. If the rate of change of the coefficients of the resulting trigonometric equation is assumed to be small in comparison with rotor frequency, a solution can be obtained by equating the summation of coefficients of similar functions to zero. Only the longitudinal equation is of interest in this study. If the case of two counter-rotating rotors or a single rotor machine with negligible coupling between lateral and longitudinal motions is considered and the pitching velocity of the tip path plane in the lateral direction is neglected

$$-(B_{1c} - B_1)(2k_2 + 2Dc_2) - B_1 \frac{C_h}{3}$$

$$= \mu_x C_h\left[-\theta_o - \lambda_a - \frac{C_{m_o}}{a h_1}\right] + \dot{\mu}_x\left[-\theta_o \Omega R 1_1 m_1\right]$$

$$+\alpha_1\left[\frac{C_h}{3}\right] - \ddot{\alpha}_1\left[m_1 l_1^2\right] + a_1\left[\frac{C_h}{3}\right]$$

$$+\dot{a}_1\left[2\Omega M_s' + 2\Omega l_1 m_1 r_1\right]$$

The equation of motion about the flapping hinge can be written

$$\mu_x M'_{y_{\mu_x}} - \alpha_1 + \dot{a}_1 M'_{y_{\dot{a}_1}} - a_1 = B_1$$

Combining these two equations gives the rotor equation for the case of viscous and elastic restraints in the control system.

$$\dot{\mu}_x\left[2c_2 M'_{y_{\mu_x}} + \theta_o \Omega R l_1 m_1\right]$$

$$+\mu_x\left[M'_{y_{\mu_x}}\left(2k_2 - \frac{C_h}{3}\right) + C_h\left(\theta_o + \lambda_a + \frac{C_{m_o}}{a h_1}\right)\right]$$

$$+\ddot{\alpha}_1\left[m_1 l_1^2\right] + \dot{\alpha}_1\left[-2c_2\right] + \alpha_1\left[-2k_2\right]$$

$$+\ddot{a}_1\left[2c_2 M'_{y_{\dot{a}_1}}\right] + \dot{a}_1\left[M'_{y_{\dot{a}_1}}\left(2k_2 - \frac{C_h}{3}\right) - 2c_2 - 2\Omega\left(M_s' + l_1 m_1 r_1\right)\right]$$

$$+a_1\left[-2k_2\right] = B_{1c}\left[2k_2 + 2Dc_2\right]$$

The other two equations of motion are equations 6.32b and 6.33b on page 229 of Reference 4.

If the pilot's control is assumed to be fixed, the rotor equation of motion can be written as

$$2B_1 k_2 + 2\dot{B}_1 c_2 = -\mu_x C_h \left[\frac{\Theta}{3} + \frac{\lambda}{3} + \frac{C_{m_0}}{a h_1} \right]$$

$$-\dot{\mu}_x \left[\Theta_0 \Omega R I_1 m_1 \right] - \ddot{\alpha}_1 \left[m_1 I_1^2 \right]$$

$$+\dot{a}_1 \left[2\Omega M_s' - \frac{16 C_h}{3\gamma_1 \Omega} + 2\Omega I_1 m_1 r_1 \right]$$

The terms proportional to pitching acceleration and translational acceleration are small and for most cases can be neglected. Dividing through by twice the spring constant k_2, an expression similar to that for an autopilot is obtained in which the feedbacks are proportional to forward velocity and pitching velocity of the tip path plane.

$$B_1 + \frac{c_2}{k_2}\dot{B}_1 = -\mu_x \frac{C_h}{2k_2} \left[\frac{\Theta}{3} + \frac{\lambda_a}{3} + \frac{C_{m_0}}{a h_1} \right]$$

$$+ \frac{\dot{a}_1}{2k_2} \left[2\Omega M_s' - \frac{16 C_h}{3\gamma_1 \Omega} + 2\Omega I_1 m_1 r_1 \right]$$

The ratio $\frac{c_2}{k_2}$ is equivalent to the characteristic time lag of the autopilot.

In most cases the feedback proportional to forward velocity is small in comparison with that due to the pitching velocity of the tip path plane. If this former feedback is neglected and if Hohenemser's "quasi static" condition is assumed, i.e. $\dot{a}_1 = -\dot{\alpha}_1$, the transfer function of the equivalent autopilot can be written as

$$\frac{B_1}{\alpha_1} = -\frac{\lambda\left[2\Omega M_s' - \frac{16 C_h}{3\gamma_1\Omega} + 2\Omega I_1 m_1 r_1\right]\frac{1}{2c_2}}{\left[\frac{k_2}{c_2} + \lambda\right]}$$

where $\frac{c_2}{k_2}$ is again the time lag. For the case of no viscous restraint in the system the equivalent time lag is zero and the transfer function becomes

$$\frac{B_1}{\alpha_1} = -\lambda\left[2\Omega M_s' - \frac{16 C_h}{3\gamma_1\Omega} + 2\Omega I_1 m_1 r_1\right]\frac{1}{2k_2}$$

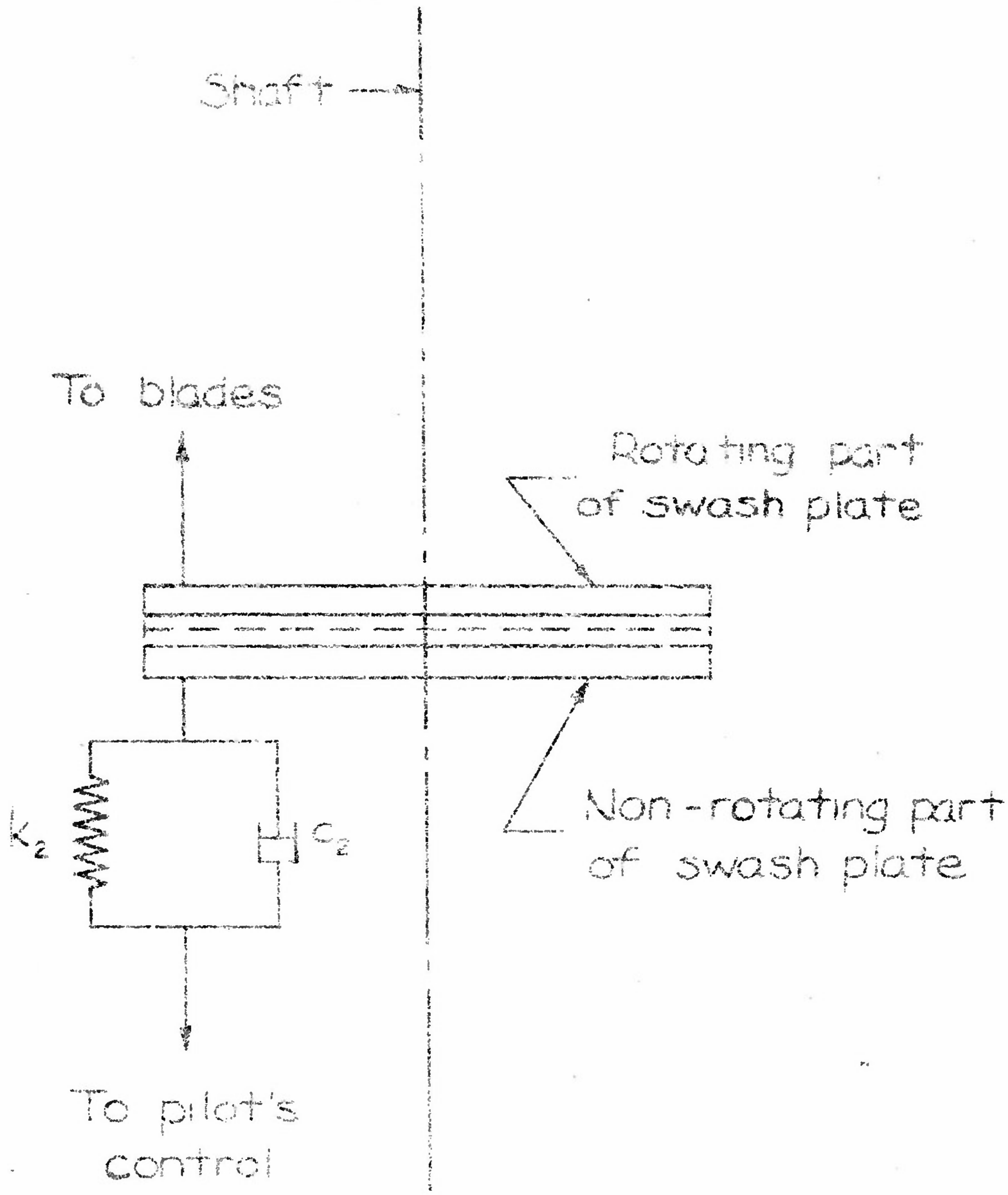

Fig. 1 Schematic Diagram of Viscous and Elastic Control Restraints

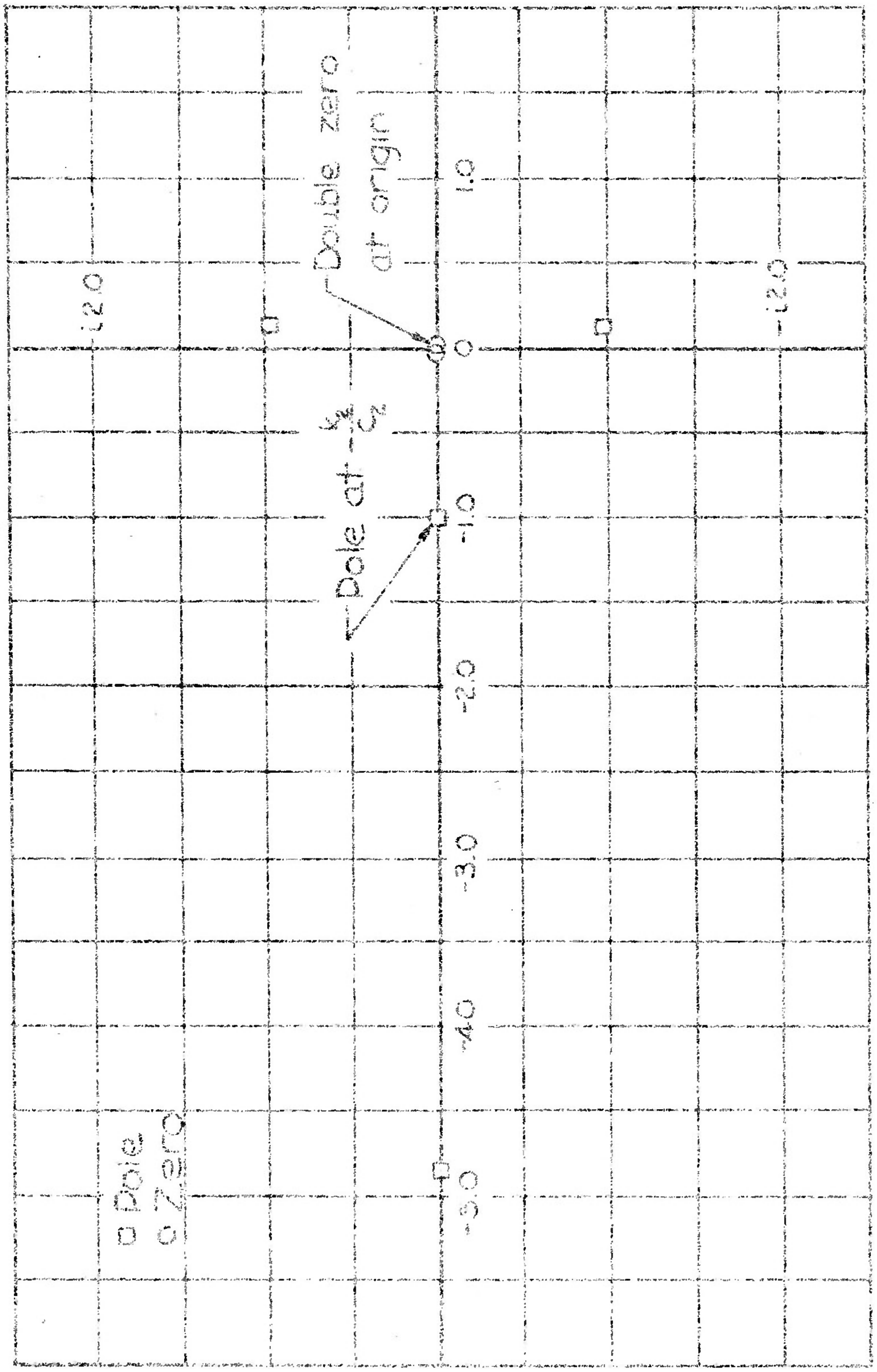

Fig. 2 Location of Poles and Zeros on Complex Plane

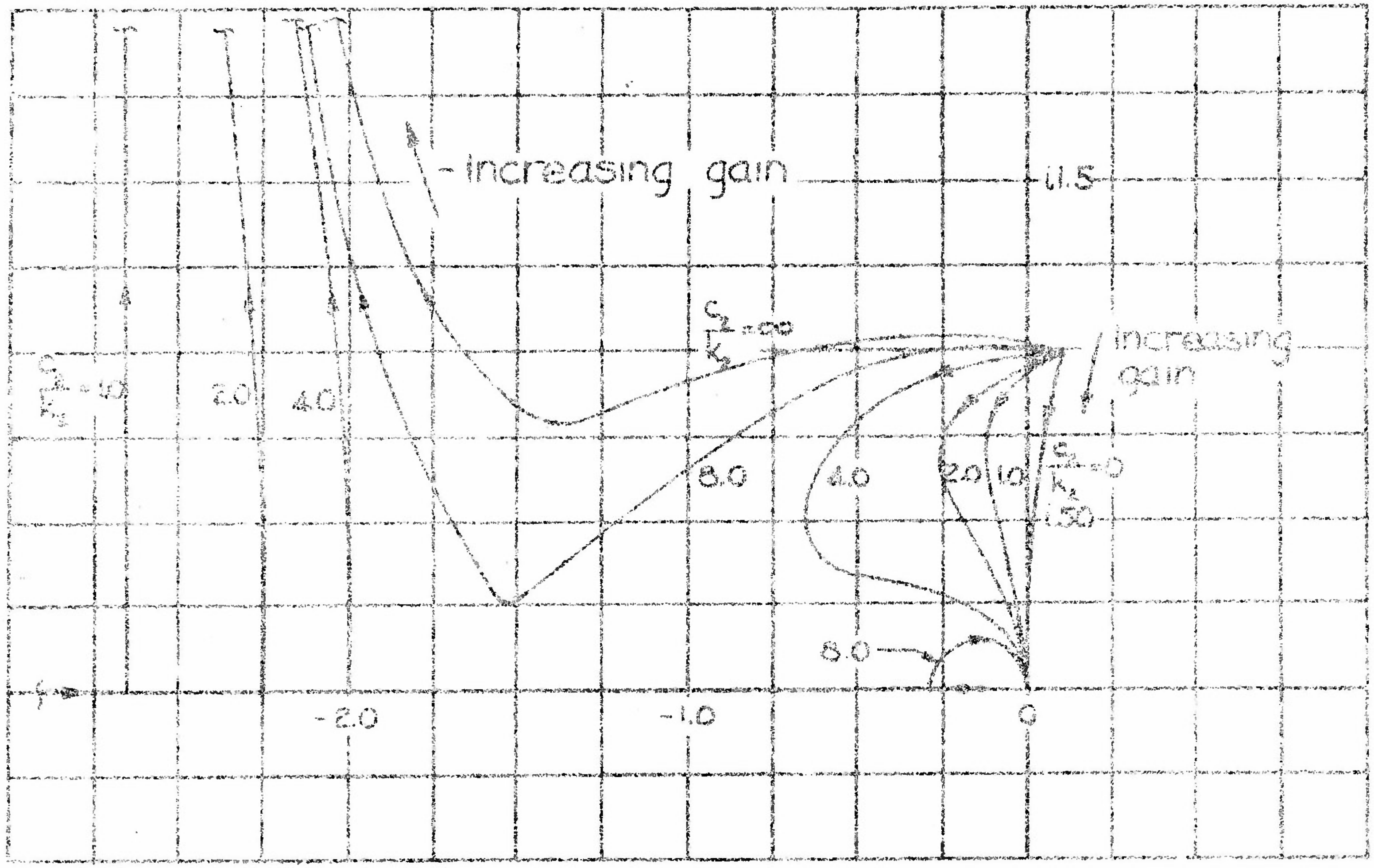

Fig. 3 Root Locus Plot of Model Helicopter with Viscous and Elastic Control System Restraints

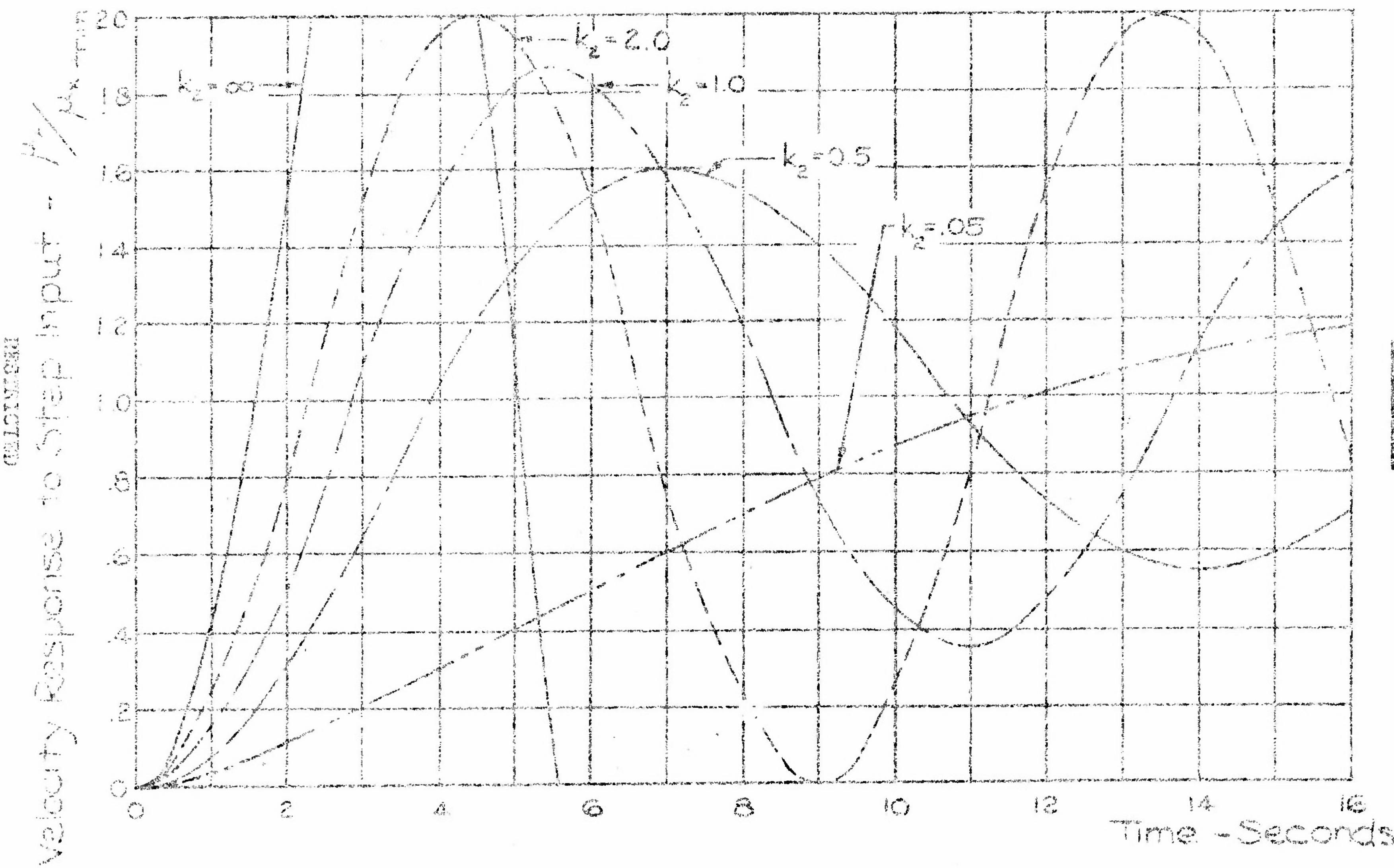

Fig. 4 Velocity Responses for Model Helicopter for c_2 = .25 and $I_1 m_1 r_1$ = .000466

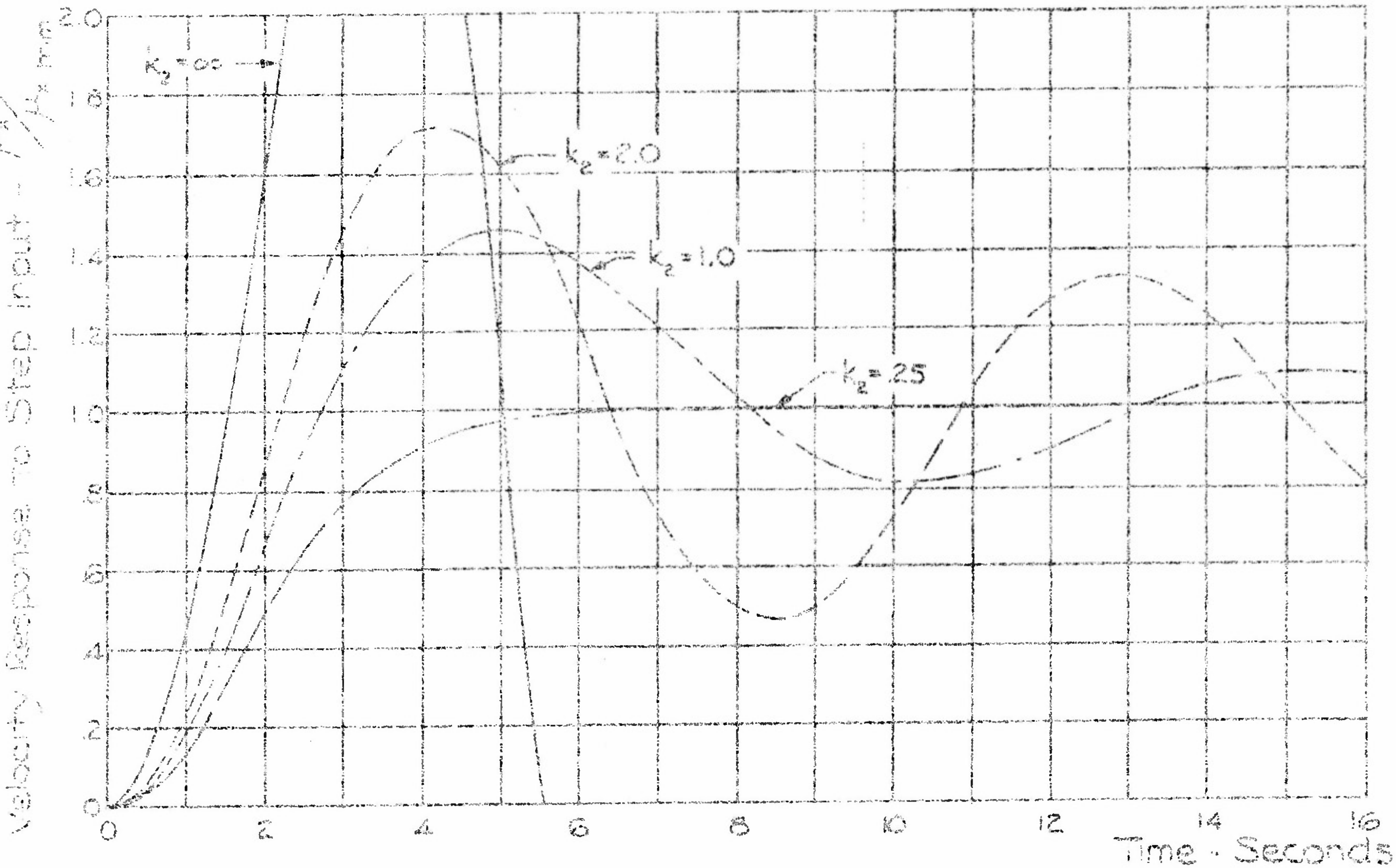

Fig. 5 Velocity Heponses for Model Helicopter for c_2 = 1.5 and $l_1 m_1 r_1$ = .000486

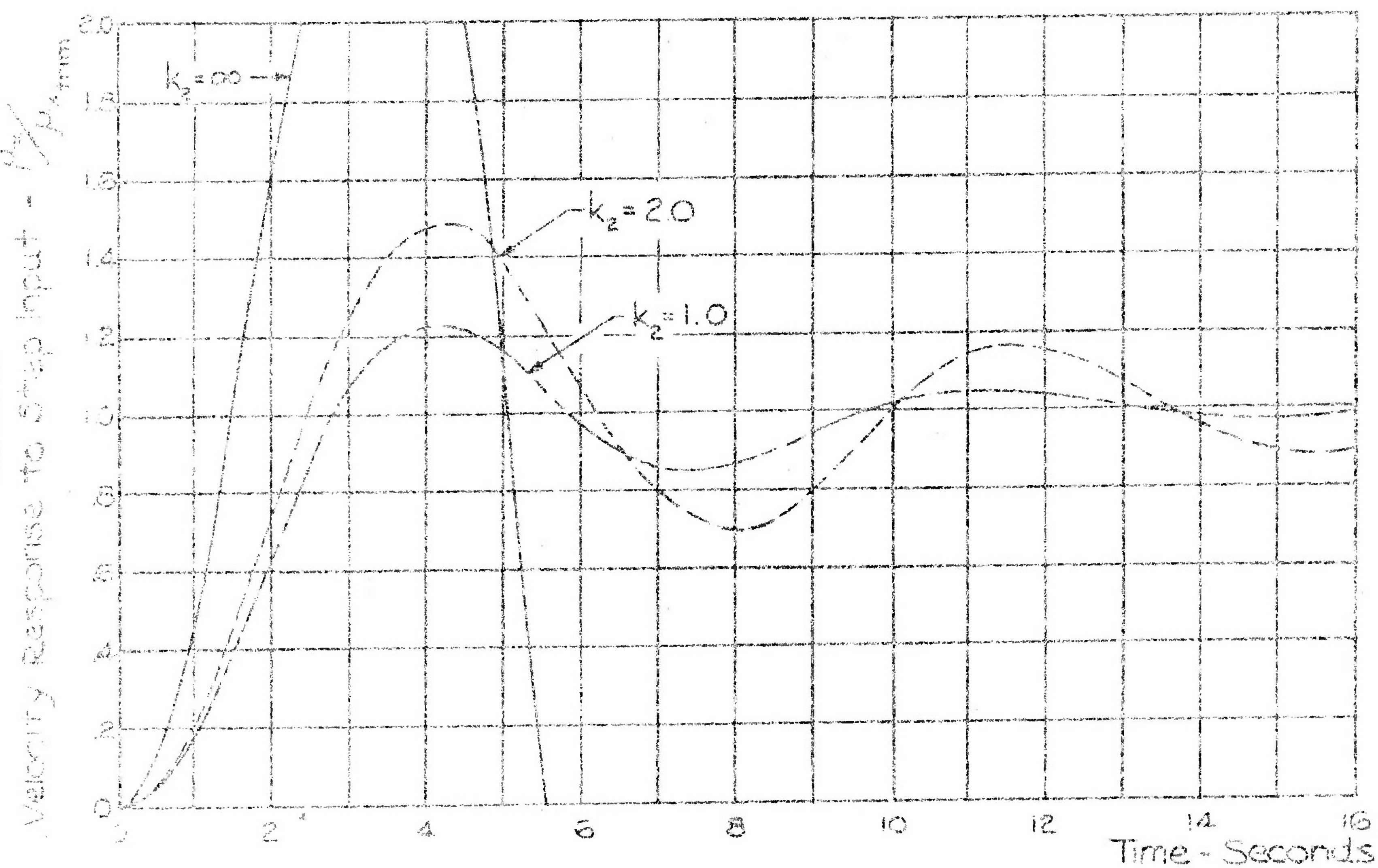

Fig. 5 Velocity Responses for Model Helicopter for c_2 = 3.5 and $l_1 m_1 r_1$ = .00486

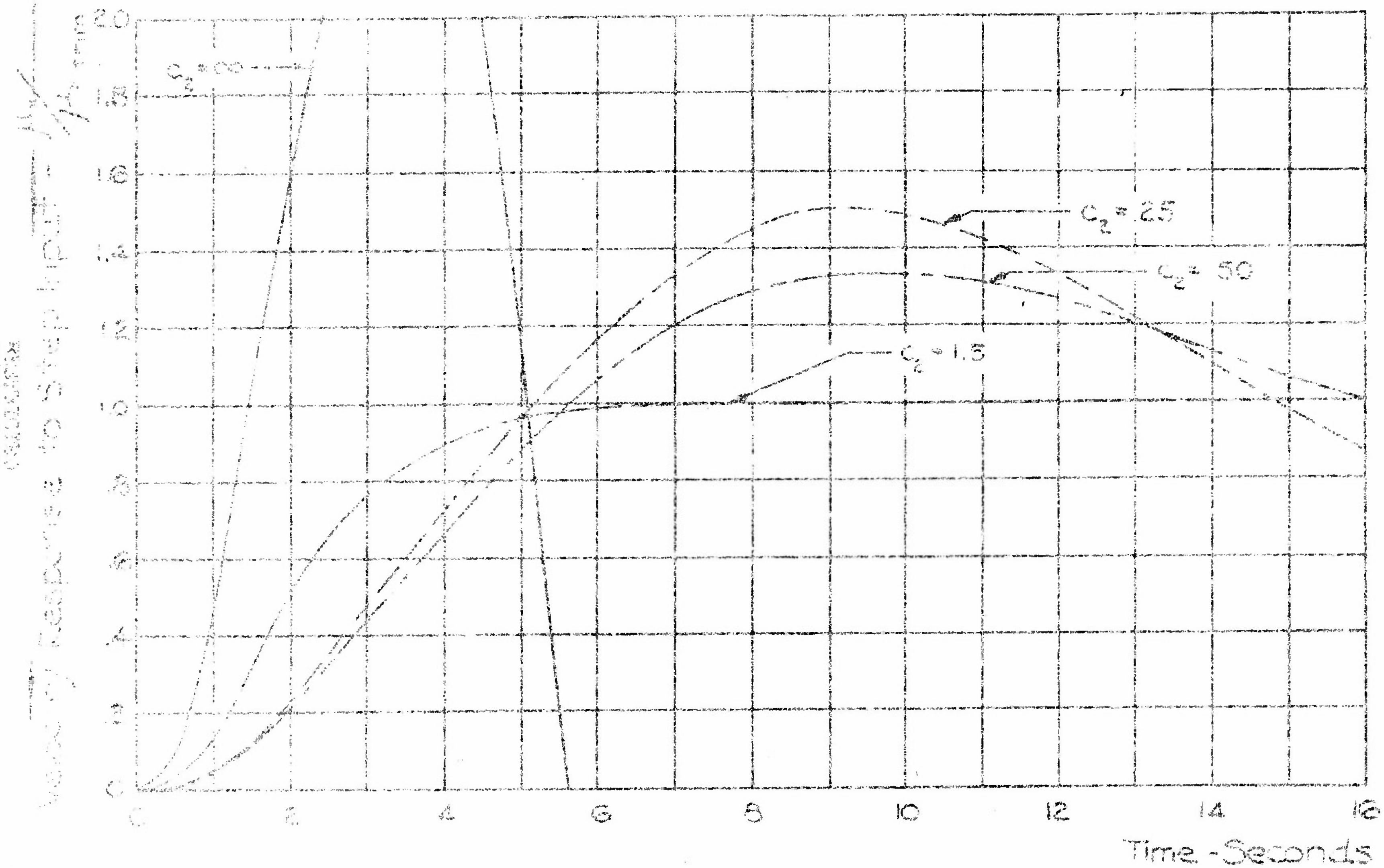

Fig. 7 Velocity Responses for Model Helicopter for k_2 = .25 and $l_1 m_1 r_1$ = .000486

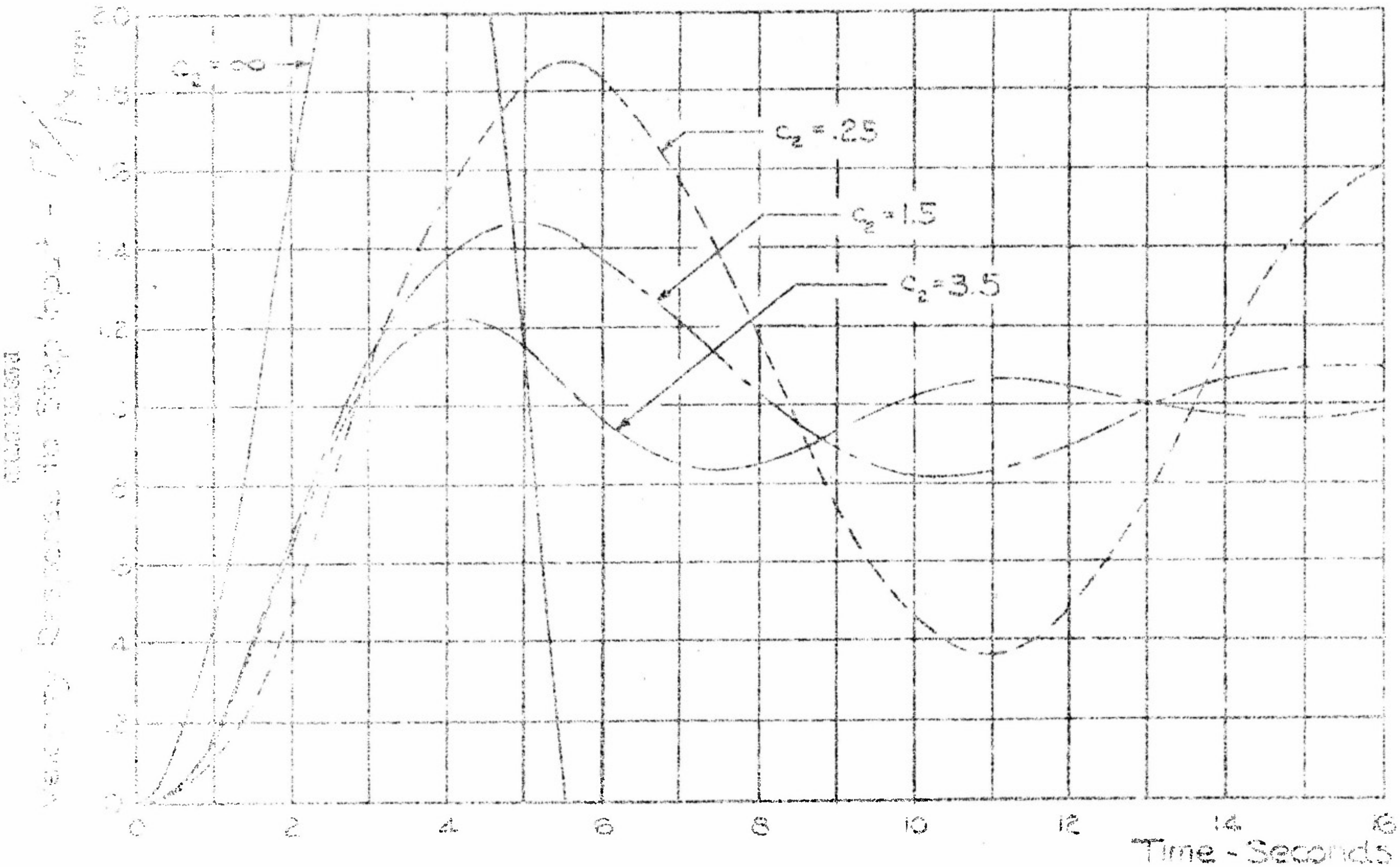

Fig. 8 Velocity Responses for Model Helicopter for k_2 = 1.0 and $l_1 m_1 r_1$ = .000486

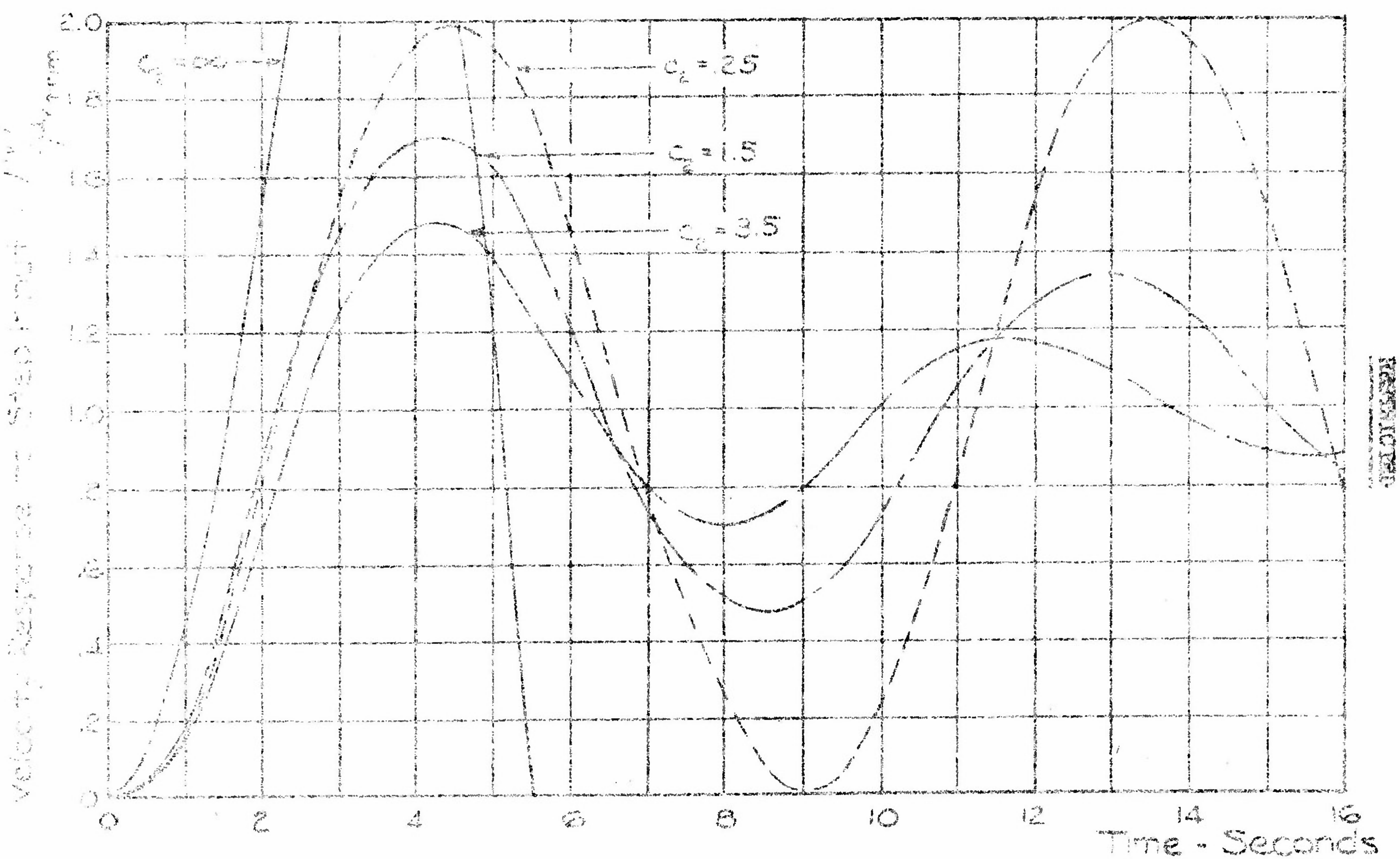

Fig. 9 Velocity Responses for Model Helicopter for k_2 = 2.0 and $l_1 m_1 r_1$ = .000486

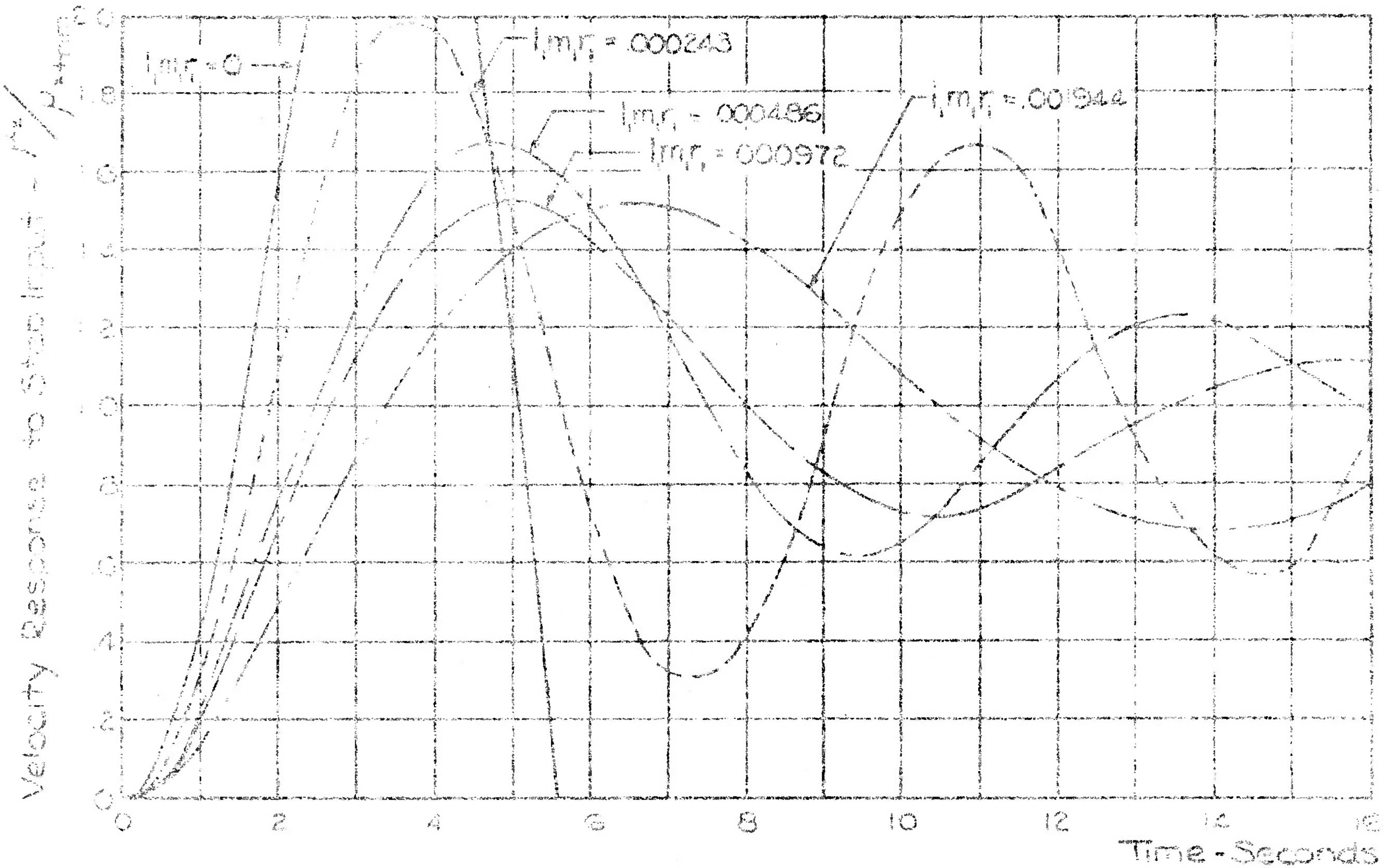

Fig. 10 Velocity Responses for Model Helicopter for $c_2 = 2.5$ and $k_2 = 2.0$

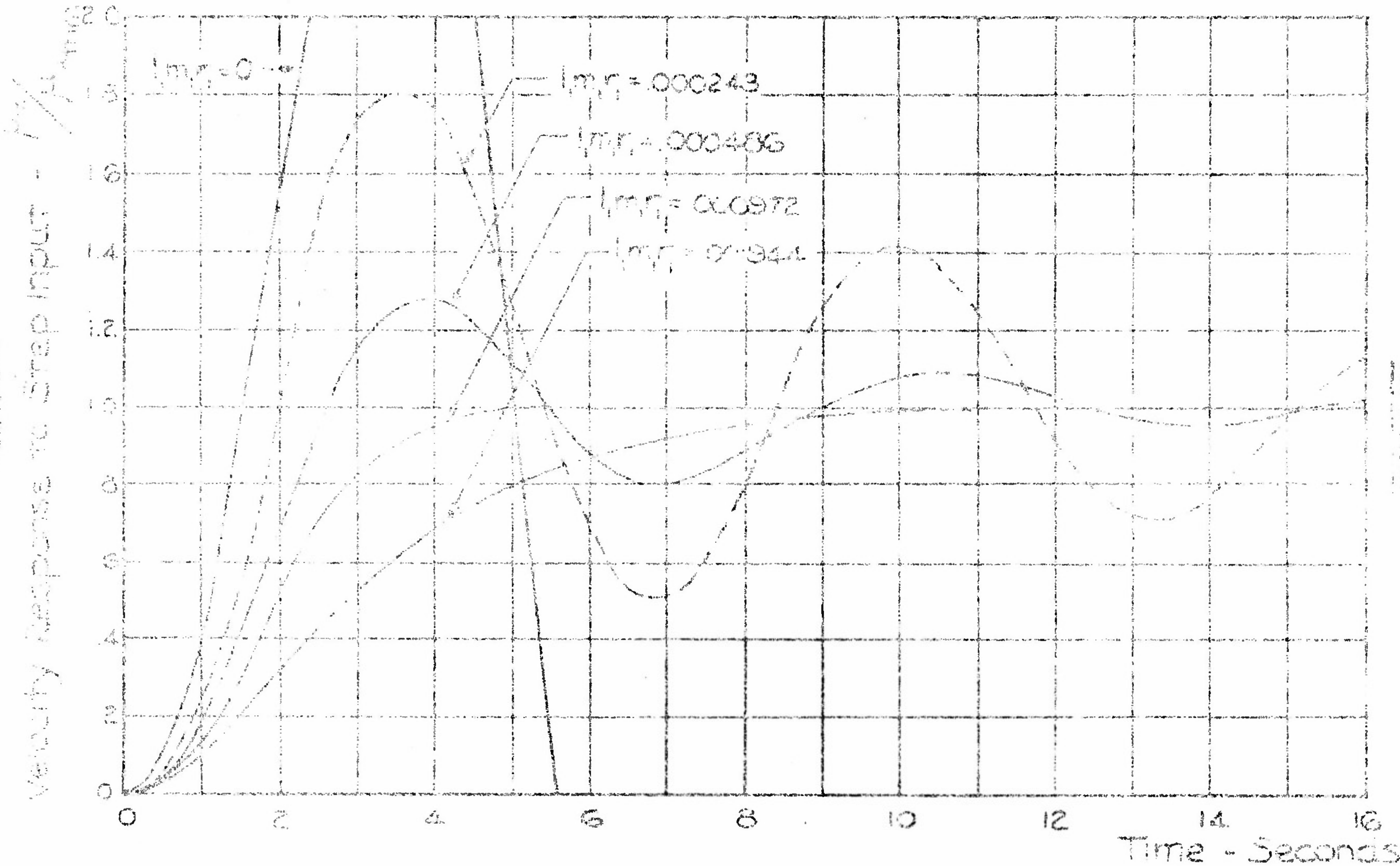

Fig. 11 Velocity Responses for Model Helicopter for $c_2 = 3.5$ and $k_2 = .50$

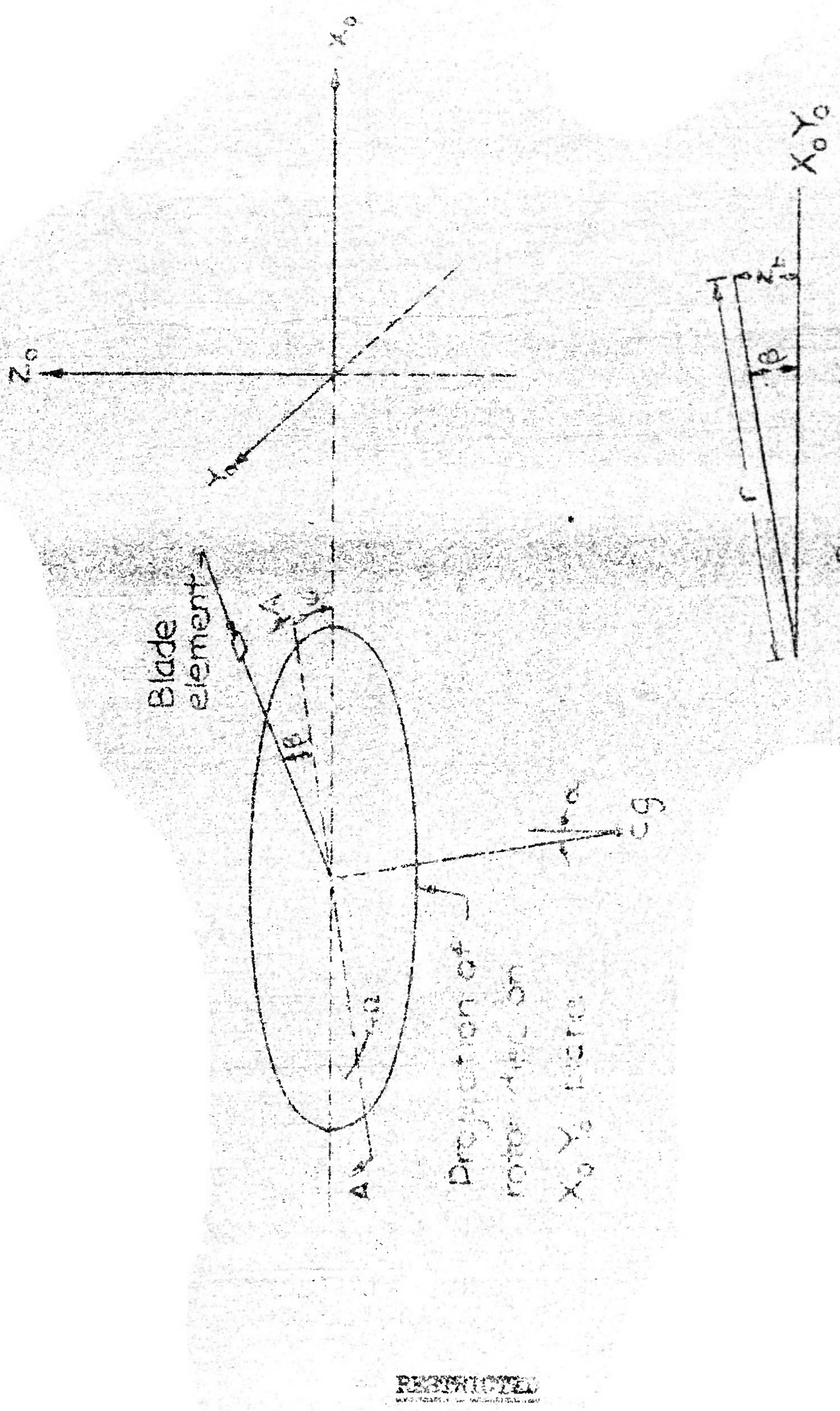

Fig. 12 Coordinate System

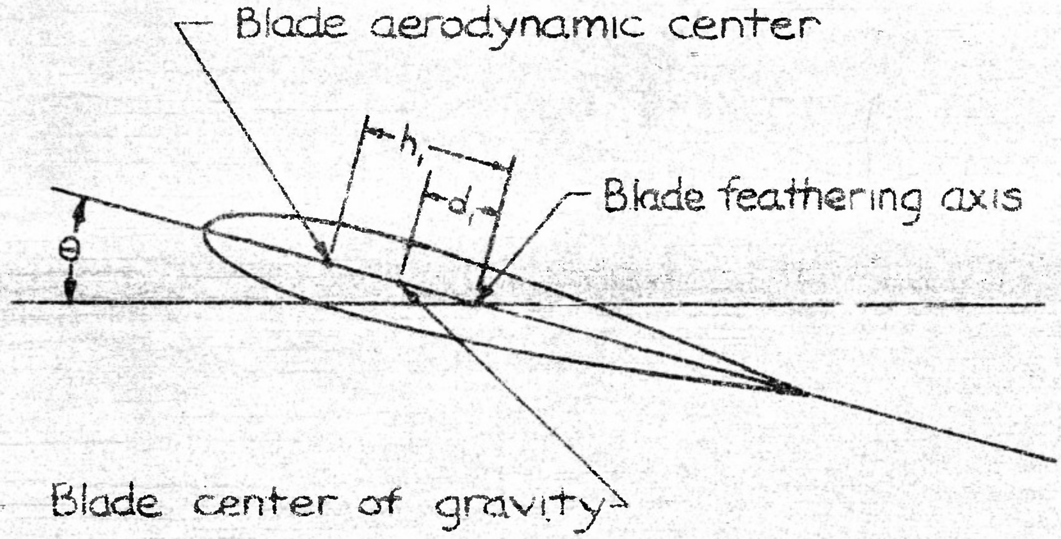

Fig. 13 Coordinate System

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